Introduction to Coordinate Graphing

CHAPTER 3

OVERVIEW

In this lesson, students learn how to use ordered pairs of numbers, called *coordinates*, to plot points on a coordinate grid. The mathematical convention for plotting points is fairly easy to explain to students. However, along with teaching the skill, this lesson helps students understand the underlying structure of the system and see how axes—intersecting perpendicular number lines—make it possible to locate points anywhere on a plane.

BACKGROUND

The system we use in this book to graph points is a mathematical invention attributed to René Descartes (1596–1650), a French mathematician, philosopher, and scientist who is sometimes called "the father of modern mathematics." Descartes's most important contribution to mathematics was his invention of analytic geometry, a way of connecting algebra and geometry that enabled mathematicians to display an equation as a set of points on a graph. Analytic geometry does more than connect algebra and geometry; that had been done by Archimedes and many others and had become the usual method of procedure in the works of the mathematicians of the sixteenth century. The great advance made by Descartes was that he saw that a point in a plane could be exactly located if its distances, say x and y, were given from two fixed lines drawn at right angles in the plane. The values of these two distances, the coordinates of a point—(x, y)—determine its location. For an equation such as $y = x + 3$, the number of solutions is infinite, and each is a pair of numbers for x and y. For example, if $x = 1$, then $y = 4$, a solution that we can represent as (1, 4). Other solutions include (2, 5), (3, 6), (5, 8), and so on. Descartes learned that when he used the solutions from equations like this one as coordinates and graphed the points, the points formed a straight line. He also discovered that when an equation involved a squared term, for example $y = x^2 + 3$, the points formed a curve. Also, for some equations containing two squared terms, for example $x^2 + y^2 = 25$, the points made a circle. From his investigations, Descartes discovered that the higher the degree of the equation (that is, the larger the exponents), the more complex the curve of its graph.

As an introductory lesson to graphing based on Descartes's work, this

lesson only introduces students to the system Descartes invented for locating points on a plane. Along with teaching students how to use ordered pairs of numbers as coordinates to locate points, the lesson gives students a background in how Descartes thought about the system.

To end the lesson, we suggest that you read aloud to the class *The Fly on the Ceiling, A Math Myth*, by Julie Glass. The story introduces René Descartes not only as a great thinker but also as a person so messy that he couldn't find a thing in his house. The story goes on to spin a yarn about how Descartes figured out his system of locating points on a coordinate grid to bring some order to his messiness. While this is an easy reader geared for grades 2–3, we found that fourth and fifth graders were completely engaged in the story and delighted by it. Our recommendation is that you read this story after the students have learned about plotting points so that the skill they've learned provides a context for enjoying the story.

To provide your students with additional practice plotting points, see Chapter 4, "Tic-Tac-Toe: Practice with Plotting Points," and Chapter 9, "Four Points: Investigating Patterns in Coordinates."

VOCABULARY

axes, axis, coordinates, graph, horizontal, intersect, number line, ordered pair, origin, plane, point, vertical

MATERIALS

- lined notebook paper, 1 sheet per student
- half-inch graph paper, 1 sheet per student (see Blackline Masters)
- *The Fly on the Ceiling, A Math Myth*, by Julie Glass (New York: Random House, 1998)
- optional: overhead transparency of lined notebook paper
- optional: overhead transparency of half-inch graph paper

TIME

- one class period

The Lesson

"How many of you sometimes wonder about things and how they came to be?" I asked the class. Several hands went up.

I continued, "I sometimes wonder, too. One thing I've wondered about is why someone invented paper with lines arranged in this way." I held up a sheet of half-inch graph paper for the class to see.

"I'm not sure why someone did it, but I know what that kind of paper is called," Brianna said. "It's graph paper."

"I've heard it called that," I said. "Has anyone heard it called other names?"

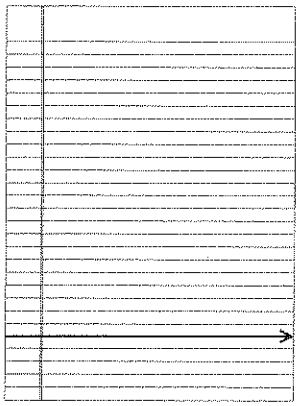
"It can be called grid paper," Jaime said.

"It's squared paper," Michael added. No one else had an idea.

"I've heard it called all those things," I said.

I then distributed lined notebook paper with a red margin ruled down the left side of the page, one sheet to each student. I said, "When I thought about paper like this, I thought about one reason that graph paper came to be. First I remembered that we can locate numbers on a number line."

On a projected overhead transparency of the lined paper, I darkened the fifth line from the bottom by tracing over it. Also, I drew an arrow at the right end of the line to indicate that it could be extended. I said, "The darkened line will be my number line."



"I can locate any number on it," I continued.

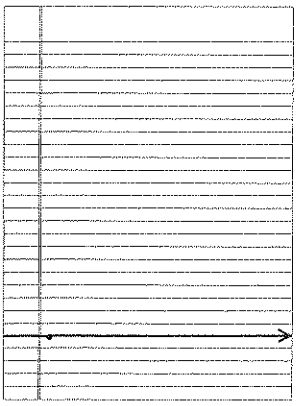
"What about a million?" Gary asked.

"Yes, I could locate one million," I said. "The arrow I drew on the end of the line means that the line goes on and on. The point for one million might be out in the parking lot."

"That's cool!" Kris said.

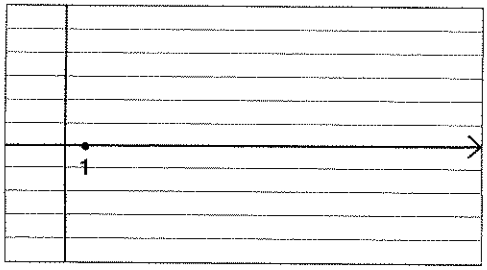
I said, "Darken a line near the bottom of your paper as I did on mine." I wanted the students to construct their own number lines rather than merely watch me demonstrate. Firsthand experience helps develop students' understanding and increases the likelihood that they'll remember what they've learned. I circulated and made a quick check around the room to see that all students had done as I had asked.

"I'm marking a point on my number line," I said as I marked a point along the darkened line about one line space away from the red margin line. "Please mark a point on your paper at approximately the same spot."



"Like this?" Audrey asked. I nodded and quickly made a visual check of the rest of the students' papers. All did it correctly except for Taylor, who put the point one line space from the left edge of the paper, not from the red margin line. I pointed this out quietly and he quickly corrected it.

"The point I just marked is where I've located the number one," I said as I labeled it.

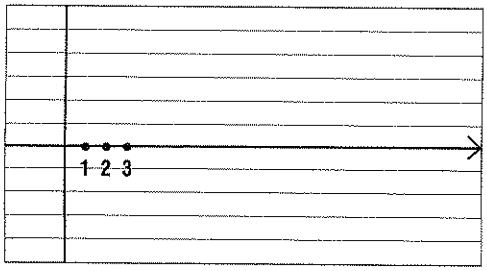


"Should we label ours too?" Sam wanted to know. I nodded.

"Now I'm going to move just a bit to the right on the line and mark another point," I said. "Don't mark your paper yet—just watch. I'll label the point *two*." I marked a point that was as far from the first point as the first point was from the red margin line. Then I marked another point one space to the right of the 2 and asked, "What number do you think I should use to label this point?"

Hands went up immediately. "Tell me in a whisper voice," I said.

"Three," the class responded. I labeled the point.



"What do you notice about how I marked the points on the number line?" I asked.

"They're not at the beginning of the line," Annie said.

"They're before the middle, but sort of in the middle," Armando added.

"The numbers go up by one," James shared.

"I spaced the points in a particular way," I said. "Who thinks you know how I spaced them?"

"Oh," Brianna said, "the space between one and two is about the same amount as the space between two and three."

"Yes, that's how I tried to space them," I confirmed." I wanted the stu-

dents to be aware that the spaces between consecutive numbers should be equal. If no student had made this observation, I would have pointed it out. I then marked a point on the darkened line near the right edge of the transparency and said, "I don't think that it would be right to label this point with the number four. Who can explain why I think this?" Several children had ideas.

"It's too far over," Gary said.

"Why do you think that?" I probed.

"Because one, two, and three come one right after the other, and you put four way down the line. It doesn't look right."

"I think it should go the same amount over," Karly said. "Then you would go one, two, three, four, all in a row." Others nodded their agreement. I erased the point I had marked on the far right.

"You're both right," I responded. "It's important on a number line for the points for numbers that go in order, like one, two, three, and four, to be the same distance from each other. When numbers are the same distance from one another, the points that represent them should be the same distance apart, too. Keep this in mind and mark points on your line for the numbers two and three, and label them as you did for the number one." As the students did this, I circulated through the class, checking their work.

"I'm using my baby finger to make the spaces about the same," Cami commented as she worked. She had accurately marked and labeled the points on her paper.

"That's a good idea," Kris said. "I have to keep erasing because the spaces aren't the same." Cami smiled as Kris used her idea to complete his number line.

I noticed that Audrey was having difficulty with the spaces on her number line and suggested she use Cami's method.

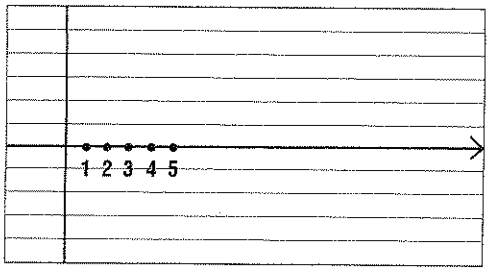
I returned to the overhead projector. After the point for the number 3 on my number line, I marked two more points, spaced the same as the first three. I pointed to the last point I marked and asked, "What shall I label this point?"

"Five," the students responded. I wrote *5* and then asked about the point before it.

"Four," the class said.

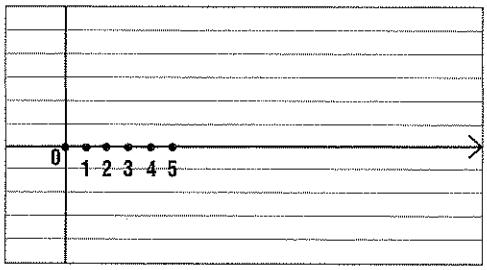
"It has to be four because the mark is about halfway between three and five and four is halfway between three and five," Michael said.

"Please mark and label points for the numbers four and five on your number line," I instructed. The students did this easily.



I then marked the point where the number line crossed the red margin. "What number should I use to label this point?" I asked.

"Zero," the class responded. I labeled the point and asked the students to mark and label *0* on their number lines.



"I can explain why it's zero," Sam said. "It's zero because it comes before one, and so does the mark we just made on the number line."

I then gave the students some historical background. "René Descartes is sometimes called 'the father of modern mathematics.' He was a mathematician who was born in France more than four hundred years ago." I stopped to write on the board:

Descartes (1596–1650)

"How old was Descartes when he died?" I asked, taking a small diversion to ask the students to calculate mentally. I waited until about two-thirds of the students had raised a hand. Then I called on Jaime.

"Fifty-four," he answered. Others agreed.

"How did you figure?" I asked.

Jaime explained, "Well, I know it was fifty years from 1600 to 1650, so I added on the four extra years from 1596 to 1600."

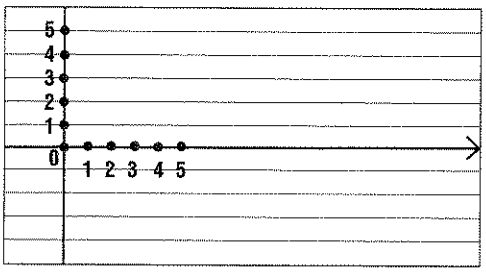
I then continued by giving the class information about Descartes. I said, "One of Descartes's mathematical inventions was to figure out a way of locating points anywhere on a plane, which is a flat surface like a piece of paper lying on your desk. Descartes's idea was to use two number lines, one going horizontally, like the one you've just made, and another going vertically. Then you use two numbers to locate a point, one telling how far to go over on the horizontal number line and the other telling how far to go up or down on the vertical number line. Every point on the plane or paper can be located exactly with two numbers. Just as an address tells exactly where you live so that the mail carrier can deliver your mail, each point on a plane in Descartes's system has an address, and each address is a pair of numbers."

"How are we going to make another number line?" Beatriz asked.

"On this paper, we'll use the red margin for a vertical number line," I said as I modeled this on the overhead transparency, drawing over the margin with a red marker. "The handy thing about lined paper is the lines are spaced equally so we can label the points where the red margin and blue lines meet, or *intersect*. Let's label some of the points on this vertical number line. I'll go up from where the two heavy lines intersect to where the

next blue line intersects the margin line, and I'll label that point *one*." I did this and waited for the children to do the same on their papers. *Intersect* was probably a new word for some of the students, but I didn't stop to define it. I find that children learn what a word means from hearing it used in context. I was careful to use *intersect, meet,* and *cross* interchangeably.

"Then we can write two, three, four, and five going up, can't we?" Annie asked. I nodded, labeled those points on the overhead transparency, and directed the children to do the same.

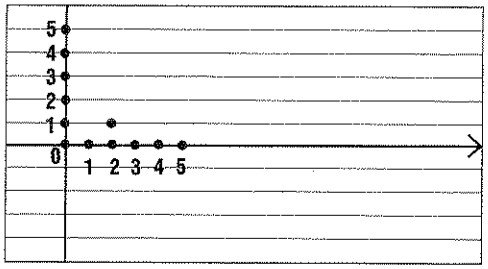


"What about this point, where the two number lines cross?" I asked, pointing to the intersection of my two number lines.

"It's zero, just like on the other line," Sam said.

"Yes," I confirmed and then added, "This point where the two number lines meet has a special name. It's called the *origin*. We use this intersection as our starting place when we locate points. Since it's located at zero on both number lines, its address is (zero, zero)." I stopped to write *origin* on the board. Beside it I wrote *(0, 0)* to model for the students how to write the coordinates in parentheses.

I then marked the point (2, 1) on the overhead transparency. "If I start at the origin and want to locate this point, how far over to the right do I need to go and then how far up?"



"It looks like you go two over," Audrey said. I placed my marker at the origin and counted two over.

"And how far up?" I asked.

"One," Jaime said. I counted one up and landed on the point I had marked.

"The address for this point is (two, one)," I said. On the board, I wrote *(2, 1)*.

"Oh, I know!" Cami said. "If you go to the point you made on the graph and put your two fingers on it and then move your fingers to the heavy lines, your fingers will come to two on the horizontal number line and one on the vertical number line." I did as Cami suggested and showed my fingers crossing the horizontal axis at 2 and the vertical axis at 1. There were several "oohs" and "ahhs" as I did this. Cami was pleased with herself.

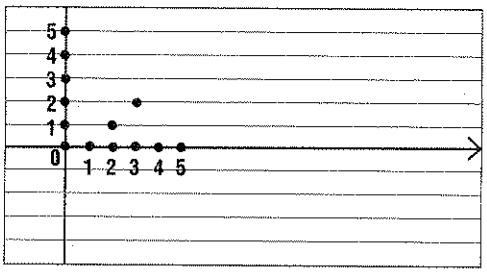
I said, "Cami's idea is one way to show why the point is located at (two, one). Watch as I show you another way to check." I put my marker on the origin and explained, "I start where the two number lines intersect, the origin. Then I count two over and one up, and I land on the point I marked." I did this, counting aloud as I moved my marker. "The point's address, (two, one), tells exactly how to get there from the origin."

"Can we mark it on our papers?" Karly asked. I nodded and checked that all of the students did this correctly.

"Here's another point," I said as I marked a point at (3, 2) on the coordinate graph. "Who knows its address?"

"(Two, three)," David said.

"I think it's (three, two)," Armando disagreed. There were no other suggestions.



"(Two, three) and (three, two) will get us to two different places on the graph," I said. "Let's check (two, three) first to see where that gets us. We start at the origin, count over to the right two, and then up three," I said as I modeled counting over two and up three on my overhead transparency. "Did we land on my point?"

"No," the class responded.

"(Two, three) didn't work, so let's try (three, two)," I suggested. "Start at the origin and count to the right three and up two."

"It worked!" several children responded.

Michael was gazing at the graph and slowly raised a hand. "I think something different," he said. "I think it's over three and up three. Can I come up and show?" I nodded and Michael came to the front of the room, put his finger on the origin where it appeared on the screen, and said, "First you count over three." He counted correctly three to the right on the hori-

zontal axis. "Then you count up." This time, Michael made an error, leaving his finger on the horizontal axis as he said, "One," and then moving up to the point as he counted, "Two, three." Michael's error is a common one that students make when learning to plot points: counting the number line as one instead of zero.

Audrey raised a hand. "You said one, but you didn't go up. See, the point is across from the two on the number line, not the three." Audrey was referring to the vertical axis. Michael was confused.

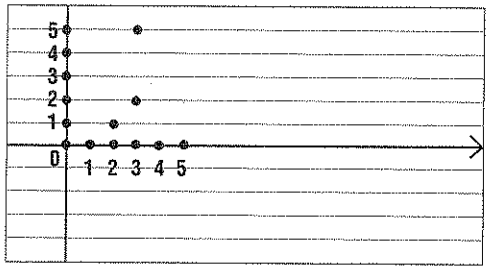
"You started correctly," I said to Michael. "Let's do it again. Start by putting your finger back on the origin." This time Michael counted over three and up two correctly, landing on the point.

"Hey, I got two up this time," he said, confused.

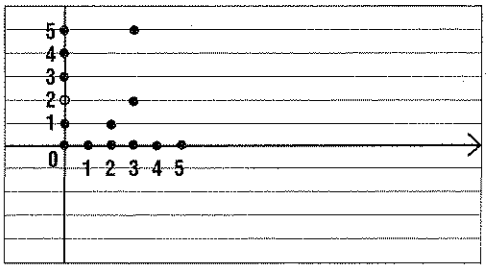
I replied, "Last time you didn't move up a space when you said 'one.' You stayed at zero." Michael returned to his seat. I made a note to myself to be sure to check back with him later. On the board I wrote *(3, 2)* and asked the students to mark the point on their papers. I checked to see that the students marked (3, 2) correctly. All but two did. I helped Natalie, and Blake got help from his neighbor, Gary.

"The order of the numbers is very important," I said. "These number pairs have a special name, *ordered pairs*, because the order is important." Under where I had written *origin* on the board, I wrote *ordered pair*. Then, underneath, I wrote *coordinates*. I explained, "*Coordinates* is another name for the pair of numbers. Each of the numbers in the pair is called a *coordinate*, and every point has two coordinates." I didn't expect children to learn these terms immediately, but I planned to use them as we continued our study. Using new terms in the context of activities is an effective way for children to learn what they mean and become comfortable using them.

I then wrote *(3, 5)* on the board. "Watch and listen to what I do as I use these coordinates to mark the next point," I said. "I start at the origin where the two number lines cross and count first to the right—one, two, three. Now I'll count up—one, two, three, four, five. Then I mark the point. This point is located at (three, five)." The students marked their papers.



I then marked the point (0, 2). "Who knows the coordinates for this point?" I asked. Hands flew up and I called on Andrew.



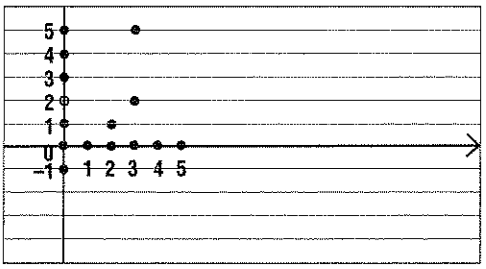
"It's nothing over and just two up," he said.

"How shall I write that?" I asked.

"Zero, comma, two," Andrew said.

I recorded *(0, 2)* and said, "We usually write the coordinates in parentheses, like this. Mark the point for (zero, two) on your paper."

I then marked a point three spaces to the right of the origin and one space down. "What do you think are the coordinates for this point?" I asked.



We hadn't studied negative numbers, but some children had heard about them and most had thought about negative numbers on thermometers. I've found that in every class in which I've done this activity, from third grade on up, there is always at least one child who has an idea about negative, or "minus," numbers. In this class, several were eager to report.

"It's three and minus one," Sam said. I recorded *(3, –1)* on the board.

"You could also say, '(Three, negative one),' " I commented. "You'll learn more about negative numbers in middle school and high school." I didn't plan to do more at this time other than establish the existence of negative numbers.

Using Graph Paper To bring the students' attention back to the usefulness of graph paper, I said, "After thinking about Descartes's system for locating points on a sheet of paper, I realized that it's much easier to plot points on graph paper than on notebook paper. Who has an idea about why this is so?" I said. Several hands went up.

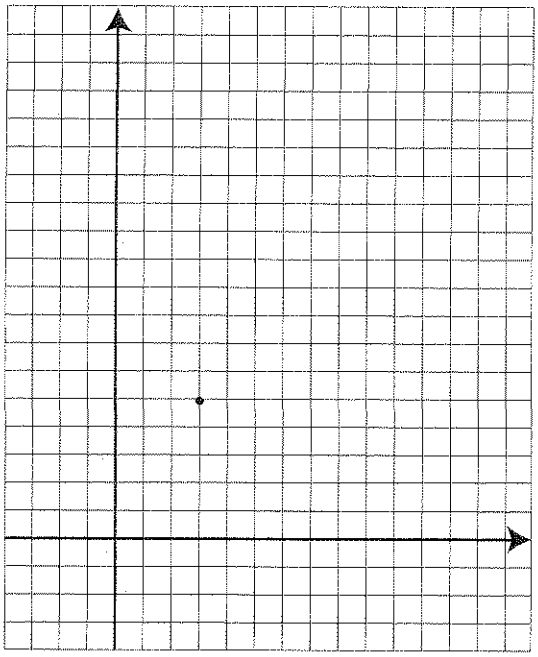
Brianna said, "It was hard to keep straight on the lined paper. With the graph paper, you have the lines going both ways."

Andrew added, "The vertical lines would really help."

Michael said, "It's hard to be exactly sure where the point is on the lined paper. You can be exact on the grid paper."

I then removed the overhead transparency of the lined paper and replaced it with a transparency of half-inch squares. "When you use graph paper to plot points, first you have to draw number lines so that you know where the origin is," I said, darkening a horizontal and a vertical line. "Each number line is called an *axis*. You have to draw the two axes." I wrote *axis* and *axes* on the board.

I pointed to the (3, 5) I had written on the board and said, "Who would like to come up and locate the point for (three, five)?" Armando came up, placed the marker at the origin, counted over three, then up five, and placed a point.



"Why does graph paper make it easier?" I asked.

"You know they're spaced right," James said.

"What's spaced right?" I asked.

"Counting over on the horizontal line. You don't have to guess," he answered.

"And it's easier to count up straight," Brianna added.

I then distributed a sheet of half-inch graph paper to each student. "First draw your two axes," I said.

"Do we write the numbers in?" Nina wanted to know.

"You don't need to," I said. "You can count fairly easily." When all of the students had drawn axes, I gave them practice plotting points. I'd write an ordered pair on the board, ask the students to plot the point on their graph paper, and then have one student come up and mark the point on the transparency. After doing this for several points, I varied the prac-

tice by marking a point on the transparency and asking the students to identify its coordinates.

Reading the Story I left time at the end of class to gather the students and read aloud *The Fly on the Ceiling,* by Julie Glass. The students were engaged and delighted by the book, and their experience learning how to plot points made the story even more enjoyable than it otherwise would have been.